2026 WAIME Problems

1. How many possible values of $a! + b! + c!$ have less than 5 digits?

2. Let $x > 1$ and $y > 1$ satisfy the system

$$\log_{10}(x^2 - 1) = \log_{10}(y^3 - 1) - 1,$$
$$\log_{10}(y^3 - 1) = \log_{10}(x^4 - 1) - 2.$$

   If $x = a\sqrt{b}$, where a and b are positive integers and b is not divisible by the square of any prime, find $a + b$.

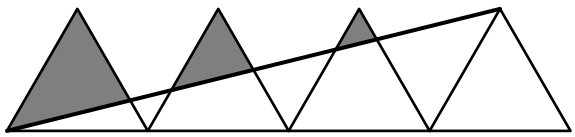
3. Kaiyuan is pouring 100 cubic units of water into a cup with shape $z = |x| + |y|$. After he pours all the water in, the water reaches height N. N can be expressed as $\sqrt[b]{a}$, where the prime factorization of a does not have a perfect power of b or higher. Find the value of $a + b$.

4. Let a decimal be "amazing" if it is between 0 and 1 and has three nonzero digits after the decimal point. For instance, 0.351 and 0.989 are amazing, but 0.705 and 1.321 are not amazing. When all amazing decimals are multiplied together, the final result has N digits after the decimal point. Find the remainder when N is divided by 1000.

5. Let a, b, and c be the roots to $x^3 - mx^2 + nx - 128$ satisfying $a < b < c$ and $b^2 - 4ac = 0$, where m and n are positive integers. What is the minimum possible value of $m + n$?

6. Non-degenerate octagon $ABCDEFGH$ with area 30 satisfies that $ACEG$ is a square and $\angle ABC = \angle CDE = \angle EFG = \angle GHA = 90^\circ$. Square $JKLM$ with side lengths parallel to $ACEG$ is constructed such that points B, D, F, and H lie on the square. The value of $\frac{[JKLM]}{[ACEG]}$ is a positive integer. Find the sum of all possible values of $[ACEG]^2$, given that $ACEG$ is inside $ABCDEFGH$.

7. 10 congruent equilateral triangles are placed side by side along a straight line, all pointing upward. A line is drawn from the leftmost bottom vertex of the first triangle to the top vertex of the 10th triangle, as shown. In the first 9 triangles, the region above the line and inside the triangle is shaded. If the shaded region has area $\frac{m}{n}$ times the total area of all 10 triangles, where m and n are relatively prime positive integers, find $m + n$.



A simplified version of this problem with 4 triangles instead of 10

8. Ryan starts with a point at $(2, 0)$ in the coordinate plane. Each turn, he rotates the point 180° about one of the points $(0, 0)$, $(1, 1)$, or $(2, 2)$. After exactly 2026 turns, the greatest possible distance between any two attainable final positions can be expressed as $a\sqrt{b}$, where a and b are integers and b is not divisible by the square of any prime, find the remainder when $a + b$ is divided by 1000.

9. Let $S = \{1, 2, \ldots, 14\}$. How many nonempty subsets of S contain no pair of integers who have a sum of 15 or a difference of 1?

10. Let t_n be a sequence defined by $t_0 = 1$, $t_1 = \frac{1}{7}$, $t_2 = \frac{1}{4}$, and

$$(4t_{n-3}t_{n-2} - 9)(4t_{n-1}t_n - 1) = 9 - 24t_{n-3}t_n.$$

Find the smallest positive integer n such that $t_n t_{n-1} < \frac{1}{3^{434}}$.

11. Let the roots of $P(x) = x^5 + 2x^4 - 3x^3 + 2x - 3$ be a, b, c, d, and e. Compute

$$\frac{1}{a^3+1} + \frac{1}{b^3+1} + \frac{1}{c^3+1} + \frac{1}{d^3+1} + \frac{1}{e^3+1}.$$

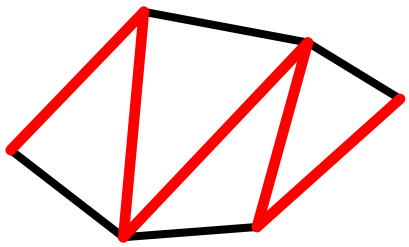
12. Find the minimum value of k such that

$$\sum_{x=1}^{k} \sum_{m=1}^{3^x} 3^m \prod_{n=1}^{3^x} \left(\left\lfloor \frac{m}{3^n} \right\rfloor - 3 \left\lfloor \frac{m}{3^{n+1}} \right\rfloor - 1 \right)^2$$

is divisible by 1000.

13. Triangle ABC satisfies $AB = 3$, $BC = 5$, and $AC = 7$. A point P lies in the interior of the triangle. Let D be the reflection of P across point B, and let E be the reflection of P across point C. Let F denote the reflection of point B across P. If F is the midpoint of segment AE, determine the square of the area of $\triangle ADE$.

14. A non-degenerate convex hexagon with area 72 has two opposite sides that are equal. Then, 3 diagonals are drawn inside the hexagon such that when combined, they connect one point of one side with the opposite point of the other side. The sum of the lengths of these diagonals and one of the equal sides equals to 24. What is the minimum integer value of the sum of squares of the other 4 sides of the hexagon?



15. A rectangle of dimensions $4\sqrt{3}$ by 4 has a point P chosen uniformly at random inside it. James folds each vertex onto P, forming four crease lines that partition the rectangle. The expected number of regions formed by those lines is $a - \frac{b\pi}{c\sqrt{d}}$ where a, b, c, d are positive integers, $\gcd(b, c) = 1$, and d is squarefree. Find $a + b + c + d$.